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NASA TECHNICAL
MEMORANDUM

NASA TM X-53026

MARCH 18, 1964

N 64 27244

Code 1 Cat. 34

NASA TM X-53026

**BOOSTER PARAMETRIC DESIGN
METHOD FOR PERFORMANCE AND
TRAJECTORY ANALYSES
PART I: CONFIGURATION**

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OTS PRICE

XEROX \$ 3.60ph
MICROFILM \$ _____

NASA-GEORGE C. MARSHALL SPACE FLIGHT CENTER

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ABSTRACT

A method is presented for mathematically describing the geometric configuration of a conventional liquid chemical booster stage for a vertically launched space vehicle. Geometric properties of all significant components were derived in parametric form. Results were summarized in schematic dimensional diagrams for two arrangements of tanked bipropellant fluids. These results will serve as a basis for formulating mass parametric equations as required for performance and trajectory analysis.

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LIST OF SYMBOLS

Input Design Parameters

<u>Symbol</u>	<u>Definition</u>
D	stage cross sectional diameter in meters
F	stage total propulsion thrust in Newtons
k_f	volume coefficient of fuel tank
k_o	volume coefficient of oxidizer tank
n	number of individual engines used in the booster mainstage propulsion system
K_{E_i}	forward bulkhead ratio of semimajor to semiminor axis of spheroid
K_{B_i}	aft bulkhead ratio of semimajor to semiminor axis of spheroid
i	subscript to imply that subscript 2 or 1 will be substituted to refer to forward or aft tank upon selection of arrangement to be investigated
r_m	propellant mass mixture ratio of oxidizer to fuel
ρ_f	fuel density in kilograms per cubic meter
ρ_o	oxidizer density in kilograms per cubic meter
W_B	mainstage propellant mass in kilograms

Dependent Design Parameters

B_i	aft bulkhead depth in meters
E_i	forward bulkhead depth in meters
H_1	axial distance of suction line from bottom of aft tank to engine gimbal plane, meters
H_2	axial distance of suction line from bottom of forward tank to engine gimbal plane, meters

LIST OF SYMBOLS (Cont'd)

Dependent Design Parameters (continued)

<u>Symbol</u>	<u>Definition</u>
ℓ_i	length of cylindrical component of propellant tanks in meters
L_t	total length of booster stage structure from forward edge of forward skirt to engine gimbal plane, meters
b_i	aft skirt length in meters
e_i	forward skirt length in meters
L_2	axial length of intertank component in meters
L_3	axial distance between tanks for access in meters
S	surface area of components identified by subsequent subscripts, meters square
V	volume of tank components identified by subsequent subscripts, meters cubed
σ	uniaxial strength of elastic materials in Newtons per meters squared

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SUMMARY

A method is presented for mathematically describing the geometric configuration of a conventional liquid chemical booster stage for a vertically launched space vehicle. Geometric properties of all significant components were derived in parametric form. Results were summarized in schematic dimensional diagrams for two arrangements of tanked bipropellant fluids. These results will serve as a basis for formulating mass parametric equations as required for performance and trajectory analysis.

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SECTION I. INTRODUCTION

Preliminary performance and trajectory studies leading to the formulation of new vehicle systems specifications depend on two very important vehicle design factors: general configuration and mass of each stage and payload. Configuration enters into trajectory studies because of its aerodynamic drag and may be readily computed by its propellant volume and by its propulsion size and arrangement. Vehicle mass, on the other hand, is not as easily estimated because of the many more constituents which must be evaluated. Vehicle design constituents which contribute to the mass of each stage are propellant as required for each mission, reserves and residuals, propulsion systems, propellant containers, thrust structures, fairings and brackets, thermal insulations, instrumentation, and staging. Since both of these vehicle design factors (configuration and mass) depend on such trajectory factors as propellant mass and flow rate, and in turn the trajectory factors depend on the vehicle design factors, an iterative process is suggested. Precious time and money are conserved when the iterative process is reserved until after interacting vehicle design equations have been applied to establish trends for optimizing stages, propellant mass, flow rates, drag, and many other significant design and trajectory parameters related to the intended missions.

Inasmuch as a knowledge of mass and relative location of each separate major functional component is not necessary for preliminary trajectory studies, the general configuration and mass equations may be developed and applied independently of the vehicle designers. Therefore, the trajectory analysts may begin their studies at once, and the vehicle designers, free from supporting trajectory studies, are allowed to restrict their early efforts to selecting materials, propulsion systems, and to optimizing shapes and component orientation. The efforts of both teams should be integrated when it appears that refinements to their studies are warranted with a minimum of iterations. This arrangement, if applied in the early study phases, will effectively compress the design time schedule and will incorporate a better combination of vehicle features.

Size and mass equations sought by the trajectory analysts involve an intelligent accumulation and interconnection of many detailed analyses which are selected from a store of subroutine equations for their parameters applicable to a particular vehicle. This, in essence, is like a mathematical "tinker toy" set. These design equations must include such parameters as diameter, liftoff acceleration, propellant mass, mixture ratio, flow rate, thrust, chamber pressure, nozzle expansion ratio, tank pressure, and staging losses.

Many preliminary analysis teams are acquiring this capability to some extent, but because of the rapid pace of our present space program, this knowledge is preceding the literature. This limited exchange of techniques is unfortunate in that these analysts are forced to initiate and develop methods separately without benefit of the experiences of others. It is therefore the purpose of this presentation to document another method developed by the author for a chemical booster stage of a vertically launched vehicle which may be extended to upper stages. This paper presents this methodology for a booster stage configuration equation. The series will continue with papers for each major component mass equation thereafter. Mathematical simplicity is essential and will be favored wherever practical, statistical data will be presented where necessary, and the International System of Units will be used throughout.

Many individuals from Aero-Astroynamics and P&VE Laboratories in the various fields of vehicle design and performance have kindly made available data from which useful range of coefficients and empirical equations were derived. Their contributions are hereby gratefully acknowledged. Thanks are due to Mr. Helmut J. Horn for his encouragement and his many valued suggestions in this paper and those to follow. Special thanks are extended to Mrs. Sarah Hightower, Mrs. Irene Dolin, and Mr. James Hackney for their assistance with the manuscript.

It is hoped that this series will serve to stimulate an interest to improve, to supplement, and to generally contribute to existing published and unpublished techniques.

SECTION II. STAGE CONFIGURATION

This part of the series treats the development of the geometrical properties of a liquid chemical booster stage for a vertically launched space vehicle. The two principal dimensions to be discussed are the stage outer diameter and lengths of all components which contribute to the total stage length.

A stage diameter is first chosen to accommodate the propellant tank requirements for optimum vehicle structure; but aerodynamic drag, dynamics, main propulsion requirements, manufacturing facilities, transportation, and launch restraints very often influence the stage final diameter. The stage diameter (D) is an input design parameter and will appear throughout the series of reports.

The stage length is dependent upon the diameter and propellant volume for tank lengths, and upon the diameter and propulsion for the thrust structure and main engine lengths, respectively.

Dimensions, surface areas, and other geometric properties will be derived in parametric form for each major component. These properties will serve as a basis for developing the mass equations in addition to the stage configuration. A schematic diagram of component arrangements and significant dimensions are shown on Figure 1.

1.0 Propellant Tanks Arrangement

A conventional liquid chemical booster requires two propellant tanks in tandem as shown in Figure 1. Either the forward or aft tank may be used as the fuel container, and the other for the oxidizer. This arrangement will depend on the optimum vehicle structure due to thrust and inertial loads, on engine suction pressure requirements, and on the minimum residual masses trapped in the suction lines and tanks at stage separation. Therefore, the tank arrangement cannot be resolved on the basis of geometry, nor does it matter in deriving the stage size or component dimensions. The arrangement will ultimately be evaluated by the performance, dynamics, and controls of the vehicle.

1.1 Propellant Tank Volumes

For a given mainstage propellant mixture ratio (r_m), the volume of each tank must accommodate reserves, residuals and transient fluids in addition to the mainstage propellant mass (W_8). Admittedly, the masses of these additional fluids are more significant to the performance analysts; however, their origin and their relative and total volumes bear some measure of consideration.

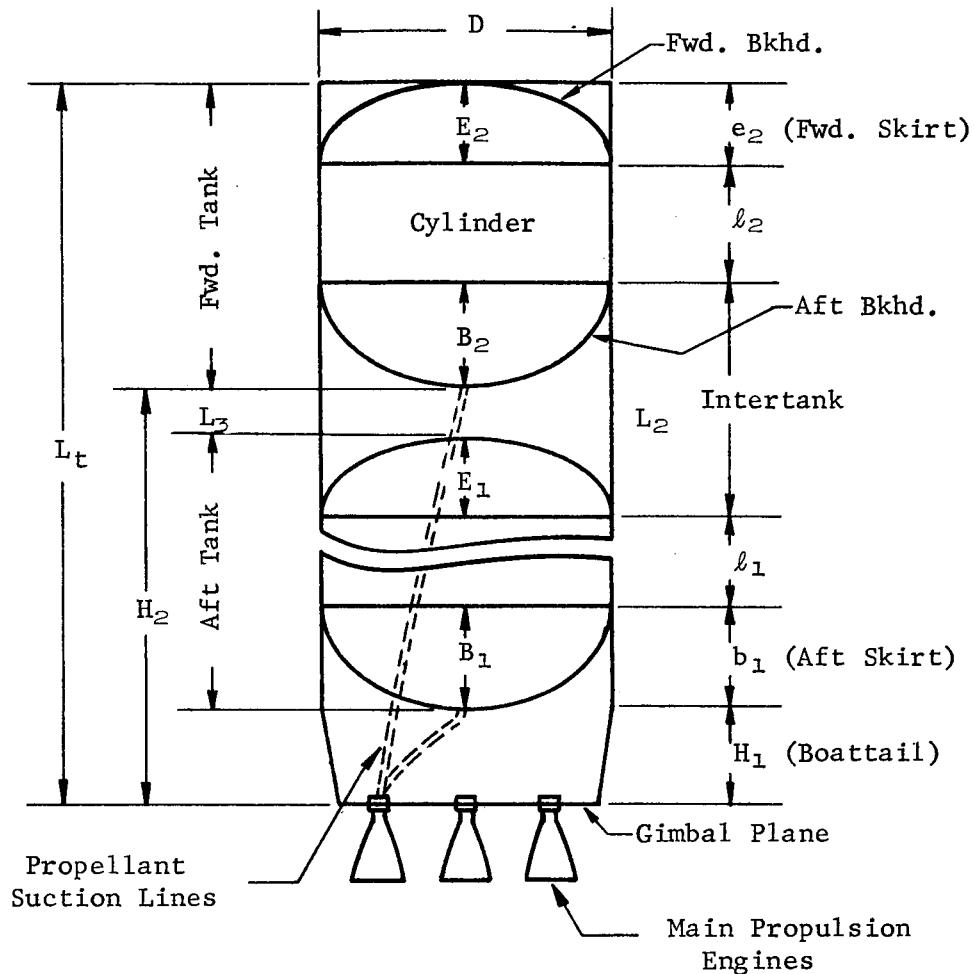


FIGURE 1. SCHEMATIC DIMENSIONAL DIAGRAM OF CONVENTIONAL LIQUID CHEMICAL BOOSTER STAGE FOR VERTICALLY LAUNCHED VEHICLE

The ullage volume is dependent upon the engine suction pressure requirements and the tanks pressurization system history. The tank arrangement and vehicle acceleration history has some influence on the ullage volume necessary. For very large stages, the ullage volume ranges between 3.0 and 5.0 percent of the total tank volume. Thrust buildup and thrust decay propellant volumes depend upon the engine characteristics and shutdown sequences. They may be conservatively related to the stage total volume as 2.0 to 3.0 percent for the first stage and 0.3 to 0.5 percent for all subsequent stages. Propulsion performance reserves consist of propellant allotments for main engine performance deficiencies due to mixture ratio shifts or specific impulse variations. This volume may be related to the total mainstage propellant volume and ranges between 0.5 and 0.6 percent. Another two percent of the total volume should be included to allow for tank thermal shrinkage, propellant topping dispersions, trapped propellants, and enclosed equipment or trespassing tunnels.

Arbitrarily combining these volumes and referring to the results as coefficients of mainstage propellants, we obtain

$$k_f = k_o = 1.10. \quad (1.1a)$$

We may now express the fuel tank volume as

$$V_f = \frac{k_f}{(1 + r_m)} \left(\frac{W_f}{\rho_f} \right), \quad (1.1b)$$

and correspondingly, the oxidizer tank volume as

$$V_o = \frac{k_o r_m}{(1 + r_m)} \left(\frac{W_o}{\rho_o} \right) \quad (1.1c)$$

where (ρ) is the density of propellants and "f" and "o" refer to fuel and oxidizer, respectively, which are presented in Figures 10 and 11.

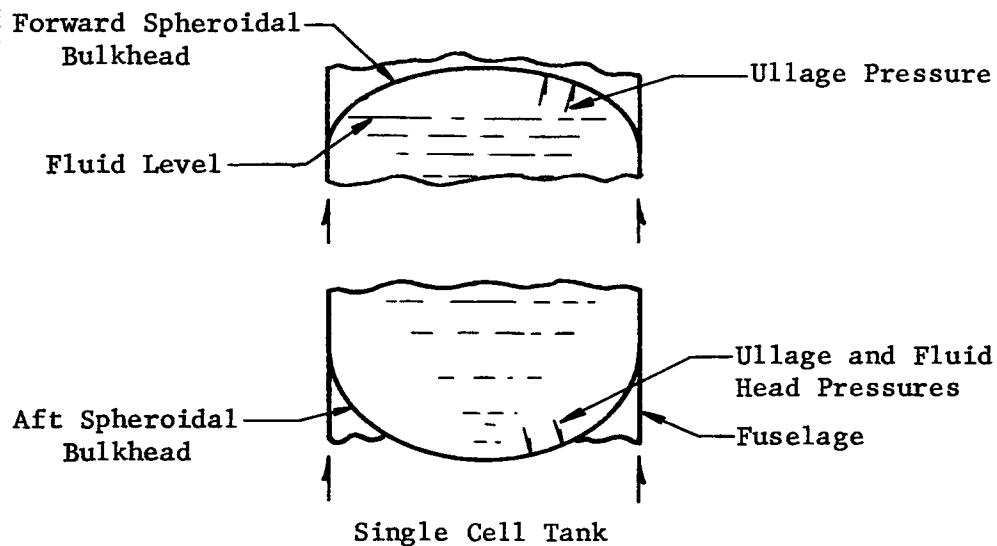
1.2 Propellant Tank Bulkhead

Propellant tank bulkheads are major components which significantly influence tank pressures, stage diameter, length, structural weight, trapped residual weights, and sloshing. The design selection of this component must therefore be reviewed by the performance, stress, and dynamics analysts, and by the fabrication technologists and cost

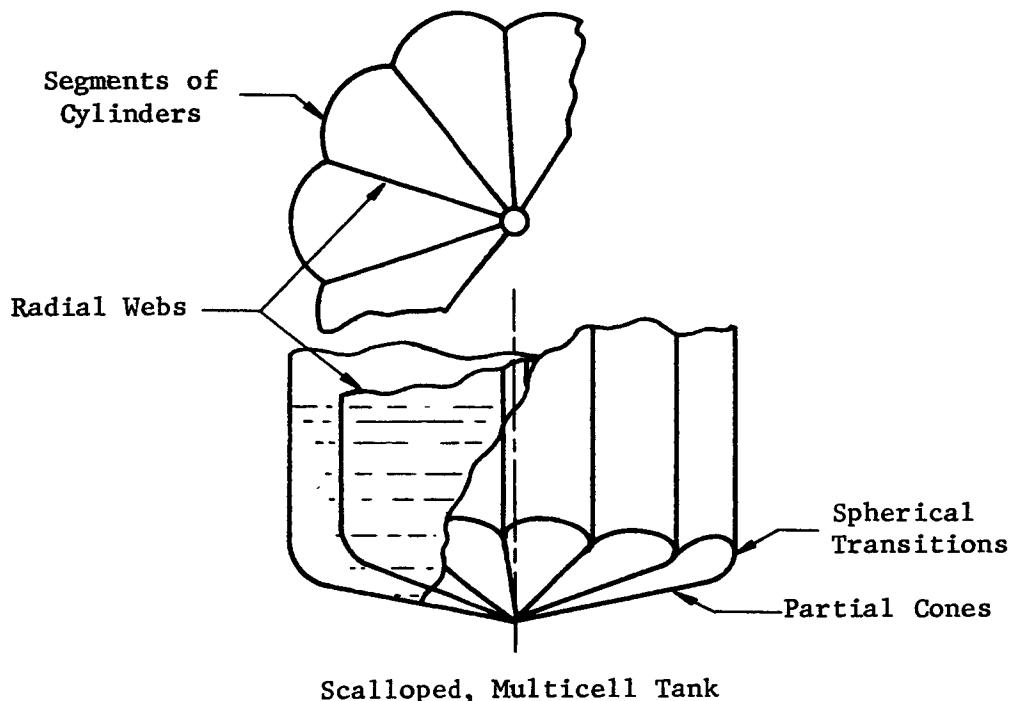
controllers. Though the scope of this presentation is to demonstrate a method of determining pertinent stage dimensions and masses required for trajectory studies, it may be fitting to discuss some interesting features of several bulkhead shapes presently used and others which have been seriously considered.

Perhaps the most commonly used bulkhead is a dome shape, symmetrical about the longitudinal axis of the vehicle, and whose meridional curve describes an ellipse or modification thereof. Imposed loads are the ullage pressure and the liquid head due to vehicle acceleration. These loads are reacted around the periphery by the fuselage structure as shown in Figure 2a. Induced stresses are essentially of the membrane type, the only structural restraints being that these membrane stresses never exceed the tensile elastic limit nor the compressive elastic stability. Because loads on the forward bulkhead are inherently less than those on the aft bulkhead, their shapes and analysis may be independently determined, fabrication cost vs design permitting. Each of these domes may be molded to a favorable strength-weight structure, mindful of the fact that increasing the dome depth increases the vehicle length and fuselage weight, but decreases the trapped liquid weights in the aft dome and decreases bulkhead shell thicknesses.

Scalloped, multicell type tanks [1], with bulkheads composed of radially intersecting partial cones with spherical transitions to vehicle cylindrical wall (shown in Figure 2b), have been investigated for large booster stage application. The radial cell walls, or webs, serve to stabilize the scalloped shape of the pressurized tank cross section as well as to couple the forward and aft bulkheads. The imposed loads are ullage pressure and liquid head under vehicle acceleration as for the dome type bulkhead. However, the reaction of the aft bulkhead load is now distributed partly to the forward bulkhead through the tank radial webs and the remainder to the vehicle fuselage. Thus, the optimum design of the forward bulkhead is dependent upon the aft bulkhead and vice versa. Because scalloping results in smaller radii of curvature, the shell thickness is less than a conventional dome type bulkhead. The mass reduction due to thinner tank outer walls must be paid back to the tank radial webs. The net result is a naturally built-in slosh baffle arrangement with very little mass increase necessary to stiffen the baffles for slosh loads. These radial webs may be further utilized in the aft tanks to partially support the inboard engine thrust. Other important advantages of multicell bulkheads over the conventional ones are that (1) skirts are considerably reduced in length which decreases vehicle size, and therefore mass, (2) the outer wall thicknesses may be adjusted by the number of cells to admit single pass welding for large boosters, and (3) the shallow bulkhead is more adaptable to large diameter-to-length tank ratio. At present, the multicell tank application is untried and the associated development problems and costs have not been fully evaluated.



(a)



(b)

FIGURE 2. SINGLE AND MULTICELL TANK BULKHEADS

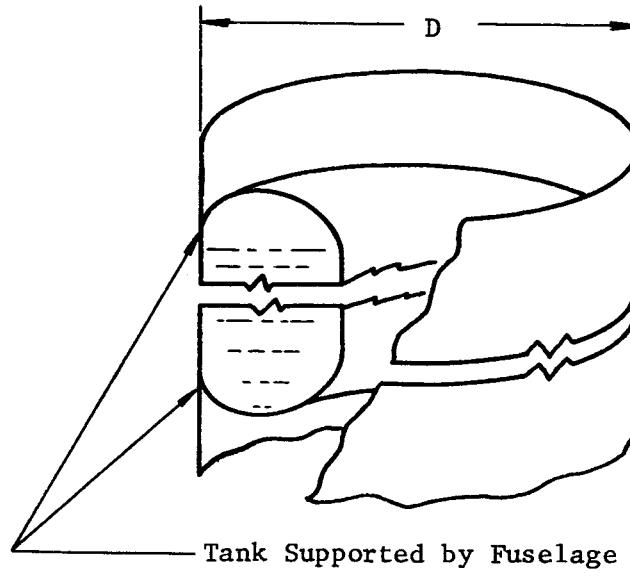
Another common candidate for large diameter-to-length type bulkhead is the torus shown in Figure 3a. Though the surface area for a given volume is greater than the two types discussed, it may be designed into any stage with substantial decrease in length by increasing the outer diameter (D). The meridional curve of the torus type bulkhead may be defined to produce maximum volume per depth, constant biaxial stress for constant thickness* or minimum depth per maximum outer diameter (D). For very small outer-to-inner diameter ratios, the bulkhead may be supported only at the periphery of the outside diameter. Because of the unsupported inner periphery, the aft bulkhead axial displacements may become excessive with increasing inertial loads and with decreasing inner diameter. This is also true of the forward bulkhead since it is coupled to the aft bulkhead. The excessive axial displacement may be overcome by providing supports or ties at the inner periphery as shown in Figure 3b. An improved application of the torus type bulkhead is that of the "semi-toroidal tank" [2] shown in Figure 5b. The bulkhead meridional curve is defined by an oblate ellipse and the inner wall is supported by a center post which also serves as the tank container and is axially supported by the thrust structure.

Other shapes of the toroidal type bulkheads proposed are combining best membrane and geometric features of the torus and dome** of Figure 5a; fabricating a torus consisting of multiple intersecting spheres of Figure 4a; intersecting a frustum of a cone with domes at each end; and a host of others familiar to the reader.

In view of the types and combinations of bulkheads possible, it now becomes obvious that configuration and mass equations of a tank are based on the selection of tank and bulkhead and the corresponding detailed subroutine analysis of each specific component. The subroutine analyses should consist of equations of length, depth, surface area, volumes, wall thickness, mass, etc., in terms of input design parameters and flight conditions. Therefore, to demonstrate this methodology, we proceed to develop these necessary parametric equations for a conventional type bulkhead and propellant tank of Figure 1. In addition to its simplicity of presentation, results obtained for the conventional bulkhead and tank may be verified by the reader's experiences and available data.

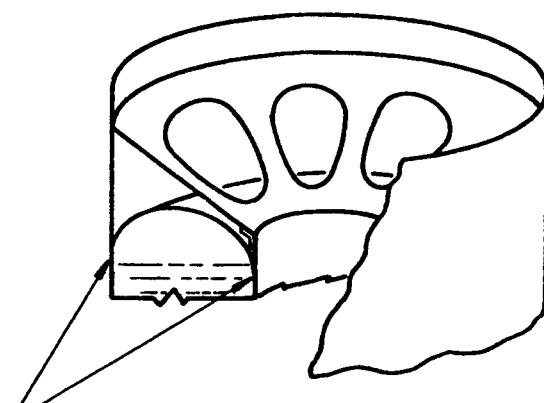
* During an informal discussion (July 1963) on large diameter-to-length ratios, Mr. H. Horn proposed that a toroidal shell could be defined for a constant shell thickness such that its membrane biaxial tensile stress is equal to the biaxial elastic strength of the material at every point. This type toroidal shell may possibly exhibit a desirable volumetric capacity per shell depth in addition to its efficient use of material.

** Suggested by Mr. H. Horn on June 1962 during informal discussion on propellant tank configuration for lunar landing vehicles.



Tank Supported by Fuselage

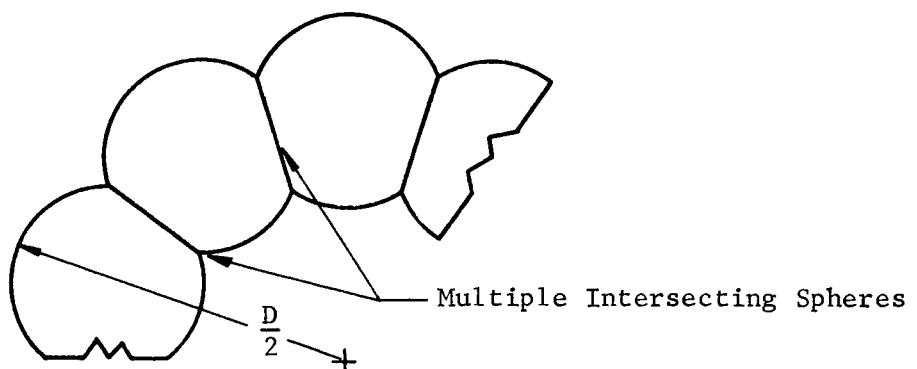
(a)



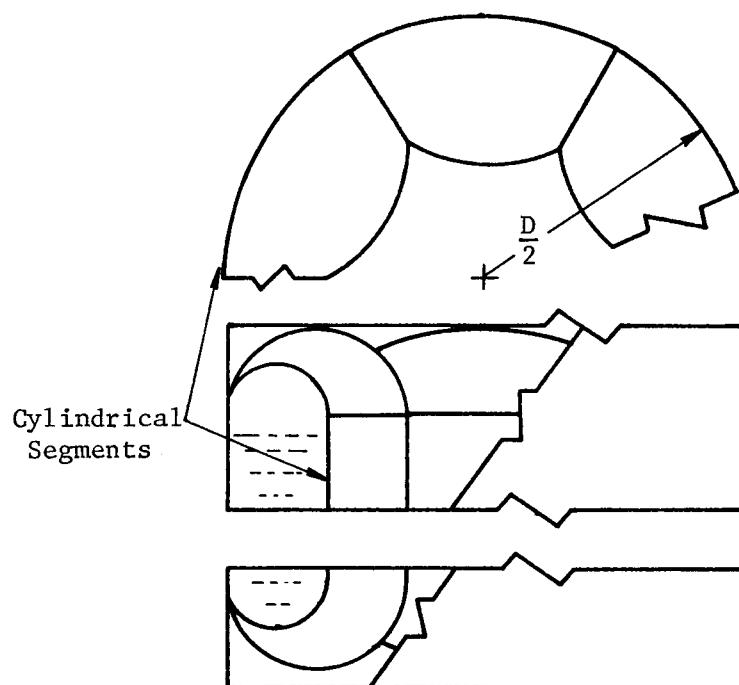
Tank Supported at Inner
and Outer Periphery

(b)

FIGURE 3. TOROIDAL TYPE BULKHEADS OF SMALL OUTER
TO INNER DIAMETER RATIOS

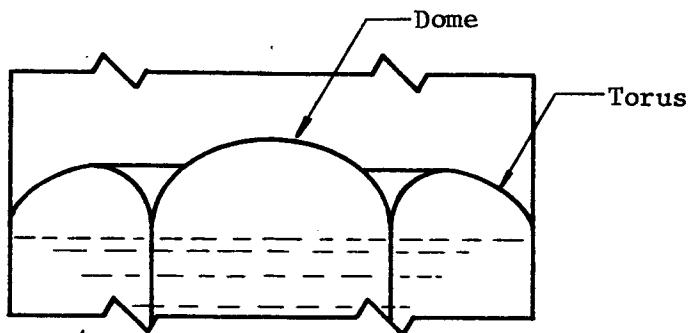


(a)



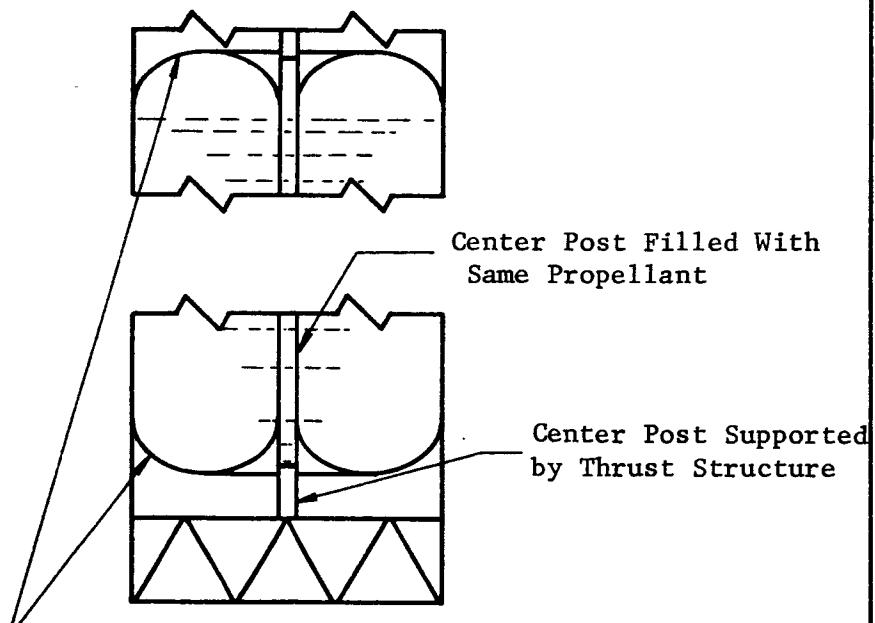
(b)

FIGURE 4. MULTICELL, TOROIDAL TYPE BULKHEADS



Combined Best Features of
Torus and Dome Bulkheads

(a)



Semi-Toroidal Bulkheads

(b)

FIGURE 5. COMBINED TORUS-CYLINDER TANKS

Assuming each bulkhead in this presentation is described by an ellipse with $D/2$ as the semimajor axis and E or B as the semiminor axis of the forward and aft bulkheads, respectively, the ratio of the axes may be expressed as an input design parameter

$$K_B = \frac{D}{2B_i} ; \quad K_E = \frac{D}{2E_i} \quad (1.2a)$$

where

$K > 1$ is an oblate spheroid,

$K = 1$ is a sphere,

$K < 1$ is a prolate spheroid,

and the volumetric capacity is

$$V = \frac{\pi}{12K} D^3. \quad (1.2b)$$

The exact solutions for each spheroidal bulkhead is given for three types:

Oblate when $K > 1$

$$S = \frac{\pi}{4} D^2 \left\{ 1 + \frac{1}{2K\sqrt{K^2 - 1}} \ln \left(\frac{K + \sqrt{K^2 - 1}}{K - \sqrt{K^2 - 1}} \right) \right\}, \quad (1.2c)$$

Hemisphere when $K = 1$

$$S = \pi \frac{D^2}{2} , \text{ and} \quad (1.2d)$$

Prolate when $K < 1$

$$S = \frac{\pi}{4} D^2 \left\{ 1 + \frac{1}{K\sqrt{1 - K^2}} \sin^{-1} \sqrt{1 - K^2} \right\}. \quad (1.2e)$$

Because of the transcendental functions in equations (1.2c) and (1.2e), future application into mass equations may prove to be very undesirable. In using the first two terms of the logarithmic and trigonometrical series into equations (1.2c) and (1.2e), respectively, approximate solutions may be obtained for $K \geq 1$

$$S = \frac{\pi D^2}{4K^2} \left\{ K^2 + \frac{4K^2 - 1}{3K^2} \right\} \quad (1.2f)$$

and for $K \leq 1$

$$S = \frac{\pi D^2}{4} \left\{ 1 + \frac{7 - K^2}{6K} \right\}. \quad (1.2g)$$

Both the exact and approximate solutions are plotted in Figure 6 for the range of K normally used in spheroidal bulkhead designs. The maximum error noted is five percent. Thus, through an admissible approximation, three equations were reduced to two. Still, these two equations imply that the oblate and prolate must be treated as two separate and distinct types of bulkheads. This condition requires that the analyst must choose one or the other from the outset. Should the optimization study indicate that a crossover is necessary, then the original surface area equation must be changed in all subroutine analyses containing it.

These discontinuities at $K \approx 1$ may be all removed by a formula approximating the exact curve of Figure 6 with a straight line shown on Figure 7. Such an empirical equation

$$S = 0.52 \frac{\pi D^2}{\sqrt[3]{K^2}} \quad (1.2h)$$

with a range of $K^2 \geq 1/4$ through $K^2 \leq 4$ was obtained as illustrated in Figure 7. The maximum error through this range is five percent. Since the surface area figures only into the mass analysis of the bulkhead shell and insulation, the error over the range of K is very insignificant. Many more of these situations will occur which will be treated in a similar manner. It will suffice to present only the resulting empirical equation and evaluate the error over the range intended.

$$\text{Half Prolate Spheroid } (K < 1): \quad S = \frac{\pi D^2}{4} + \frac{4K}{1 - K^2} \sin^{-1} \sqrt{1 - K^2}$$

$$\text{Half Sphere } (K = 1): \quad S = \frac{1}{2} \pi D^2$$

$$\text{Half Oblate Spheroid } (K > 1): \quad S = \frac{\pi D^2}{4} + \frac{8K}{\sqrt{K^2 - 1}} \ln \left(\frac{K + \sqrt{K^2 - 1}}{K - \sqrt{K^2 - 1}} \right), \quad \text{where } K = \frac{D}{2B}$$

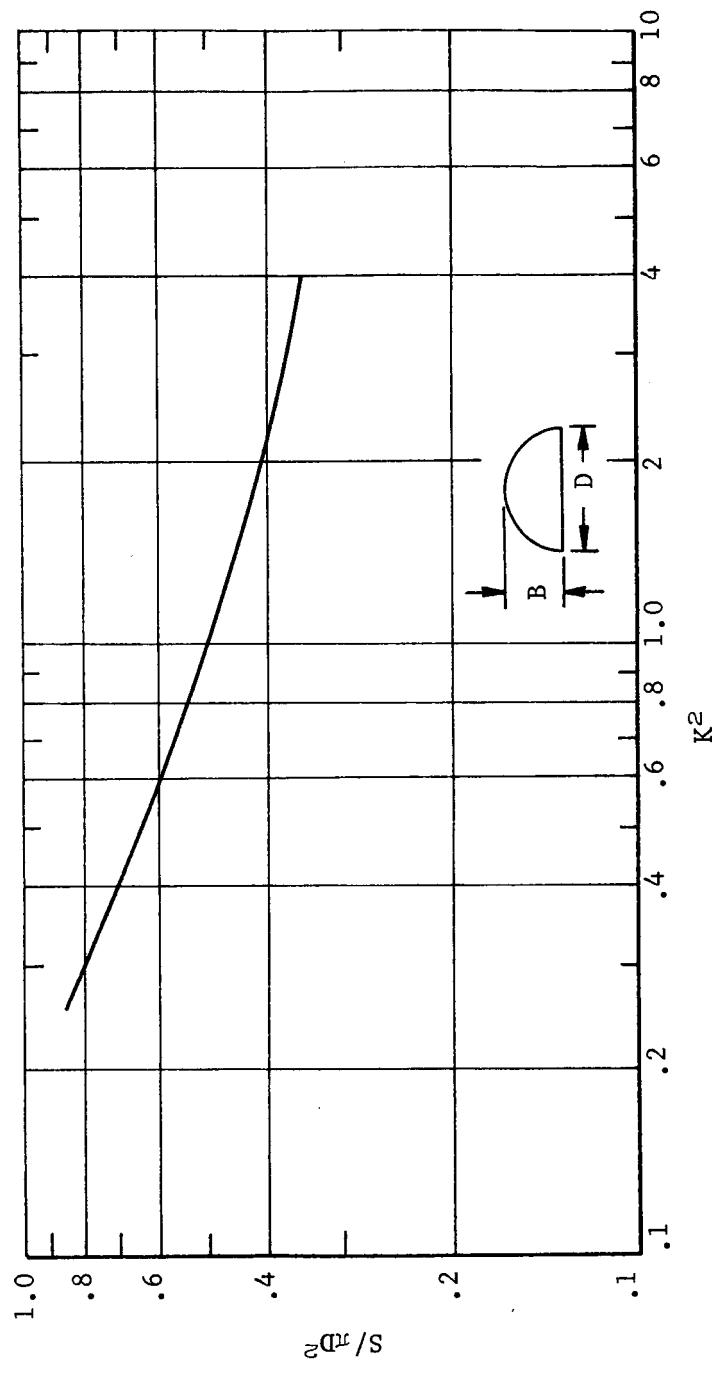


FIGURE 6. SURFACE AREA OF SPHEROIDAL BULKHEAD

The straight line approximation is defined as $\log \frac{S}{\pi D^2} = \log y + m \log K^2$, where the slope m is determined to be $-1/3$. Substituting and transforming, we get $\frac{S}{\pi D^2} = y(K^2)^{-1/3}$.

From this plot, $K^2 = 1$ when $S/\pi D^2 = 0.52$. Therefore, $y = 0.52$ and straight line approximation is $S = 0.52 \pi D^2 (K^2)^{1/3}$.

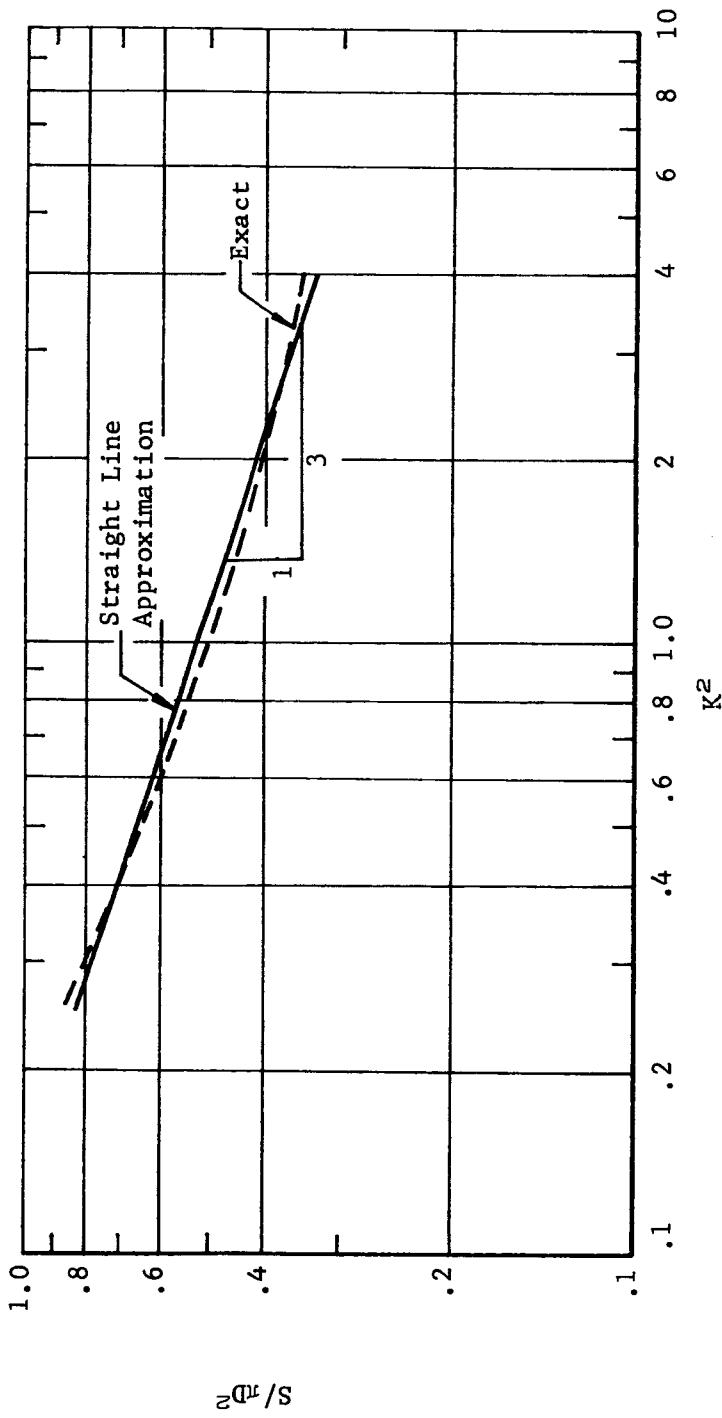


FIGURE 7. SURFACE AREA OF SPHEROIDAL BULKHEAD, EMPIRICAL METHOD

It may now be concluded that a conventional spheroidal type bulkhead has four geometric properties of interest for determining stage size and weight: a diameter (D), which is an input design parameter; a depth (B or E), which may be determined from equation (1.2a) for an optimized or selected parameter (K); a volumetric capacity expressed by equation (1.2b) in (D) and (K) parametric terms; and finally, the dome surface area of the empirical equation (1.2e) in the same parametric terms as the volume.

1.3 Propellant Tank Cylinder

As in the propellant tank bulkheads, the geometric properties of the connecting propellant cylindrical tank to be derived in parametric form are the volumetric capacity, surface area, and length. The first two properties are related to the input design parameter (D) and to the property of length (ℓ).

The primary purpose for considering the volume of the cylinder

$$V = \frac{\pi}{4} D^2 \ell \quad (1.3a)$$

is so that the cylinder length (ℓ) may be determined.

It must be noted that length (ℓ) is a dependent variable. Once the bulkhead parameters K_B and K_E have been selected for a tank, the length (ℓ) then depends solely on the required tank volume of equations (1.1b) or (1.1c) and the stage diameter (D). Having solved for the tank cylindrical length, we express its surface area in the already familiar terms

$$S = \pi \ell D. \quad (1.3b)$$

1.4 Fuel and Oxidizer Tank Geometric Properties

From the relationships presented in paragraphs 1.1, 1.2, and 1.3, it is now possible to obtain the tank composite properties from the sum of each of the component properties developed. Actually, the only combined geometric property required is the composite tank volume which is equated to the volume of the mainstage propellant to obtain the dependent length (ℓ) of the cylindrical tank section.

Adding, algebraically, cylindrical section and bulkhead volumes, using equations (1.2b) and (1.3a), we obtain the composite tank volume

$$V = \frac{\pi}{12K_E} D^3 + \frac{\pi}{4} D^2 \ell + \frac{\pi D^3}{12K_B},$$

which may be reduced to

$$V = \frac{\pi}{4} D^2 \left[\frac{D}{3} \left(\frac{1}{K_E} + \frac{1}{K_B} \right) + \ell \right]. \quad (1.4a)$$

At this point, it must be noted that each component of the vehicle must be identified in accordance with the schematic diagram of Figure 1 and also with the associated fuel or oxidizer in the case of a tank.

Commencing with the fuel tank, the fuel volume of equation (1.1b) is equated to the tank volume of (1.4a),

$$\frac{k_f}{(1 + r_m)} \frac{w_s}{\rho_f} = \frac{\pi D^2}{4} \left[\frac{D}{3} \left(\frac{1}{K_{E_i}} + \frac{1}{K_{B_i}} \right) + \ell_i \right]^*,$$

to solve for the tank cylinder length

$$\ell_i = \frac{4}{\pi} \frac{k_f}{(1 + r_m)} \frac{w_s}{\rho_f D^2} - \frac{D}{3} \left(\frac{1}{K_{E_i}} + \frac{1}{K_{B_i}} \right). \quad (1.4b)$$

Since length (ℓ_i) cannot be a negative quantity, the tank will consist of an aft and forward dome only for a mainstage propellant mass of

$$w_s = \frac{\rho_f}{k_f} (1 + r_m) \frac{\pi}{12} D^3 \left(\frac{1}{K_{E_i}} + \frac{1}{K_{B_i}} \right). \quad (1.4c)$$

* The subscript "i" must be replaced with subscript 1 or 2 to signify that the fuel tank is aft or forward, once the tank arrangement for investigation has been selected.

If the propellant quantity is less, then the tank diameter (D) or the dome parameter (K) must be adjusted; otherwise, the ullage volume of the fuel tank (which is included in k_f) must be increased to allow for this tank offloading. If the tank offloading imposes excessive structural and residual mass increases, then another tank configuration must be investigated and the associated equations derived.

Substituting equation (1.4b) into (1.3b), we write the expression for the surface area of the fuel tank cylindrical section as

$$S_{E_i} = 4 \frac{k_f W_8}{\rho_f (1 + r_m) D} + \frac{\pi D^2}{3} \left(\frac{1}{K_{E_i}} + \frac{1}{K_{B_i}} \right). \quad (1.4d)$$

The depth and surface area equations are

$$E_i = \frac{D}{2K_{E_i}} \quad (1.4e)$$

and

$$S_{E_i} = 0.52 \sqrt[3]{\frac{\pi D^2}{K_{E_i}^2}} \quad (1.4f)$$

for the forward bulkhead, and

$$B_i = \frac{D}{2K_{B_i}} \quad (1.4g)$$

and

$$S_{B_i} = 0.52 \sqrt{\frac{\pi D^2}{K_{B_i}^2}} \quad (1.4h)$$

for the aft bulkhead.

Since equations (1.4e) through (1.4h) as presented are not distinguished between forward or aft tanks until the subscript 2 or 1, respectively, has been assigned, they may equally apply to the oxidizer tank bulkhead in this general form and will not be repeated.

Similarly, the cylinder length of the oxidizer tank is derived by equating (1.1c) with (1.4a)

$$\ell_i = \frac{4}{\pi} \frac{k_o r_m}{(1 + r_m)} \frac{1}{\rho_o D^2} W_B - \frac{D}{3} \left(\frac{1}{K_{E_i}} + \frac{1}{K_{B_i}} \right) \quad (1.4j)$$

for $\ell_i \geq 0$.

Again, the conditions discussed for the fuel tanks apply to the oxidizer tank when

$$W_B < \frac{\rho_o (1 + r_m)}{k_o r_m} \frac{\pi}{12} D^3 \left(\frac{1}{K_{E_i}} + \frac{1}{K_{B_i}} \right). \quad (1.4k)$$

Using equation (1.4k) with (1.3b), we write the equation for the surface area of the oxidizer tank cylindrical section as

$$S_{\ell_i} = \frac{4k_o r_m}{\rho_o (1 + r_m)} \frac{W_B}{D} + \frac{\pi D^2}{3} \left(\frac{1}{K_{E_i}} + \frac{1}{K_{B_i}} \right). \quad (1.4m)$$

Having determined the significant properties of all the tank components in equations (1.4b) through (1.4m), we proceed to develop the remaining components of the fuselage.

2.0 Intertank Geometry

The length (L_2) and the surface area (S_{L_2}) are the only two intertank geometric properties of consequence for the configuration and mass analysis. The length is composed of the forward bulkhead depth (E_1) of the aft tank, the aft bulkhead depth (B_2) of the forward tank, and an access space (L_3) between the two bulkhead vertices. Unlike the bulkhead depths determination discussed in paragraph (1.2), the length (L_3) must be based solely on the minimum access room required between tanks for fabrication, inspection, or repair. This length must be deduced from experienced vehicles and is suggested that the empirical form

$$L_3 = 0.06 \frac{D}{2} \left[K_{E_1} + K_{B_2} \right] \quad (2.0a)$$

be used for access spaces between conventional bulkheads. Notice that, as the bulkhead shape approaches that of a prolate spheroid ($K < 1$), the access space required is naturally less. On the other hand, as the diameter (D) increases, the required space increases.

Adding lengths of equations (1.4e), (1.4g), and (2.0a), we obtain the desired intertank length equation in simplified form:

$$L_2 = \frac{D}{2K_{E_1} K_{B_2}} \left[K_{B_2} (1 + 0.06 K_{E_1}^2) + K_{E_1} (1 + 0.06 K_{B_2}^2) \right]. \quad (2.0b)$$

Using length (L_2), the intertank surface area is readily found to be

$$S_{L_2} = \frac{\pi D^2}{2K_{E_1} K_{B_2}} \left[K_{B_2} (1 + 0.06 K_{E_1}^2) + K_{E_1} (1 + 0.06 K_{B_2}^2) \right]. \quad (2.0c)$$

3.0 Skirt Geometry

Stage forward and aft skirts are those fuselage components which serve to envelop the bulkheads at the two extreme ends of the propellant tank assembly. Their lengths are identical to those of the enclosed bulkhead depths, which have already been derived. Presenting them with their proper identification as indicated in Figure 1, they are

$$e_1 = E_2 = \frac{D}{2K_{E_2}} \quad (3.0a)$$

and

$$b_1 = B_1 = \frac{D}{2K_{B_1}} \quad (3.0b)$$

for the forward and aft skirts, respectively.

Skirt surface areas are given by

$$S_{e_2} = \frac{\pi D^2}{2K_{E_2}} \quad (3.0c)$$

for the forward skirt and

$$S_{b_1} = \frac{\pi D^2}{2K_{B_1}} \quad (3.0d)$$

for the aft skirt.

4.0 Boattail Geometry

The stage boattail, as here defined, is that component between the aft skirt and the engine gimbal plane. It is approximately cylindrical in shape and its length depends upon the propulsion engine arrangement, the thrust structural requirements aft of the bulkhead vertex, and upon the propulsion system accessories.

When the propulsion system consists of multiple engines, all arranged around the boattail periphery (no inboard engines), the boattail length need only satisfy the fuel suction line routing to the engines. Since the aft skirt may be prudently substituted for the boattail, this case will not be considered further.

When the propulsion arrangement consists of a single engine system or a multi-engine system having one or more inboard engines, the boattail length is primarily dependent upon the thrust structural requirements. Assuming that the thrust structure behaves as a beam* or truss member, then the moment developed is directly proportional to the engine thrust (F/n) and the beam length (D). The moment due to thrust of a single engine (F/n) applied at the center of the structural member is also a reasonable approximation of the moment due to two (or four) inboard engines mounted around the center. Recalling that the moment is likewise directly proportional to the material strength (σ) and the beam depth cubed (H_1^3), we equate these two moments,

$$D \frac{\bar{F}}{n} = k_1 \sigma H_1^3,$$

where k_1 and σ are combined into one constant of proportionality to get

$$H_1 = \frac{1}{128} \left\{ D \frac{\bar{F}}{n} \right\}^{1/3}. \quad (4.0a)$$

* See Appendix.

Using the boattail length (H_1) we obtain the surface area

$$S_{H_1} = \frac{\pi}{128} D^{4/3} \left(\frac{F}{n} \right)^{1/3}. \quad (4.0b)$$

5.0 Propellant Suction Lines

Perhaps the most interesting feature of propellant suction lines in a tandem tank arrangement is their role in the determination of tank pressures and in satisfying pump suction requirements. Recalling that the hydraulic head contribution to the total pump suction head is a function of the propellant density (ρ) and the flight histories of vehicle acceleration and propellant level to the pump suction, then we see that the selection of lighter (or denser) fluid into the forward tank may admit a decrease in ullage pressure, thereby decreasing structural and pressurant masses. Because this phenomenon is an important variable for consideration in early trajectory studies, the axial distance between tank bottom to the pump suction flange (assumed to be at the gimbal plane) will be determined.

For the case of the aft tank suction line, the axial distance required is identically that of the boattail length (H_1). This suction line length is fixed by equation (4.0a) and is independent of the tank arrangement.

However, this is not true of the forward tank suction line, and therefore the line axial length (H_2) must be defined for the fuel and for the oxidizer in the aft tank. To find the axial length of the forward tank suction line, we simply add the lengths of the boattail (H_1), the space between forward and aft tank (L_3), and the aft tank length ($B_1 + l_1 + E_1$). The aft tank length depends on the fluid contained. Using equations (4.0a), (2.0a), (3.0b), (1.4e), and (1.4b), we express the suction line length for the case of fuel in the aft tank as

$$H_2(f) = \frac{1}{128} \left[D \frac{F}{n} \right]^{1/3} + 0.06 \frac{D}{2} \left[K_{E_1} + K_{B_2} \right] + \frac{D}{6} \left[\frac{1}{K_{E_1}} + \frac{1}{K_{B_1}} \right] + \frac{4}{\pi} \frac{k_f}{(1 + r_m) \rho_f} \cdot \frac{W_B}{D^2}. \quad (5.0a)$$

Using the above referenced equations with the exception of (1.4j) for (1.4b), we express the suction line length for the case of oxidizer in the aft tank

$$H_2(0) = \frac{1}{128} \left[D \frac{\bar{F}}{n} \right]^{1/3} + 0.06 \frac{D}{2} \left[K_{E_1} + K_{B_1} \right] \\ + \frac{D}{6} \left[\frac{1}{K_{E_1}} + \frac{1}{K_{B_1}} \right] + \frac{4}{\pi} \frac{k_o r_m}{(1 + r_m)} \cdot \frac{1}{\rho_o} \cdot \frac{W_B}{D^2}. \quad (5.0b)$$

6.0 Stage General Configuration

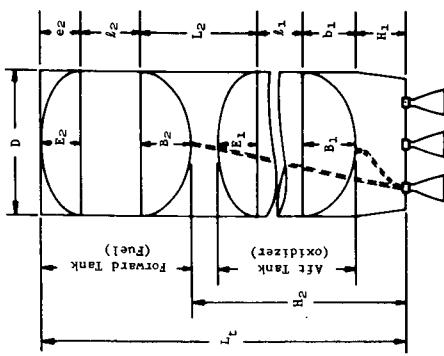
Though a vehicle booster stage is composed of many more significant components, those discussed here were primarily selected because their interconnecting geometries appropriately defined the stage configuration. Pertinent geometric properties of these components were expressed in parametric form. By methodically adding parametric equations of components, the results were a booster stage configuration in parametric form as summarized in Figures 8 and 9.

Parameters used were either of the input type or the dependent type. Parameters such as densities, propellant mixture ratio, stage diameter, etc., which must be judiciously controlled by the analysts, were called input design parameters. Those which were satisfied by connecting relations to existing conditions and input parameters were called dependent parameters. An example of this type was the length (ℓ_i) of the propellant tank cylinder.

In referring to Figures 8 and 9, we observe that by varying input design parameters, individually or simultaneously, the booster stage geometry will vary and this variation will be reflected throughout the vehicle length. Certainly, a variation in the input design parameter (D) will affect every dimension indicated in Figures 8 and 9.

Using these same parameters, equations of surface areas and volumes anticipated for use in the mass analysis were derived. Their application could be extended to mass centers of gravity and mass moments of inertia of each component for future preliminary studies of vehicle control and dynamics.

COMPONENT	LENGTH	SURFACE AREA
BOATAIL	$H_1 = \frac{1}{128} \left\{ D \frac{\pi}{n} \right\}^{1/3}$	$S_{H_1} = \pi \frac{D^{4/3}}{128} \left\{ \frac{\pi}{n} \right\}^{1/3}$
AFT SKIRT	$b_1 = B_1$	$S_{B_1} = \frac{\pi D^2}{2K_{B_1}}$
AFT BULKHEAD (AFT TANK)	$B_1 = \frac{D}{2K_{E_1}}$	$S_{B_1} = 0.52 \frac{\pi D^2}{3K_{B_1}}$
CYLINDER SECTION (AFT TANK)	$L_1 = \frac{4k_o r_m W_B}{\pi(1+r_m)\rho_D D^2} - \frac{D}{3} \left(\frac{1}{K_{E_1}} + \frac{1}{K_{B_1}} \right)$	$S_{L_1} = \frac{4k_o r_m W_B}{(1+r_m)\rho_D} + \frac{\pi D^2}{3} \left(\frac{1}{K_{E_1}} + \frac{1}{K_{B_1}} \right)$
FORWARD BULKHEAD (AFT TANK)	$E_1 = \frac{D}{2K_{E_1}}$	$S_{E_1} = 0.52 \frac{\pi D^2}{3K_{E_1}}$
INTERTANK	$L_2 = \frac{D}{2K_{E_1} K_{B_2}} \left[K_{B_2} (1 + 0.06 K_{E_1}^2) + K_{E_1} (1 + 0.06 K_{B_2}^2) \right]$	$S_{L_2} = \frac{\pi D^2}{2K_{E_1} K_{B_2}} \left[K_{B_2} (1 + 0.06 K_{E_1}^2) + K_{E_1} (1 + 0.06 K_{B_2}^2) \right]$
AFT BULKHEAD (FORWARD TANK)	$B_2 = \frac{D}{2K_{B_2}}$	$S_{B_2} = \frac{0.52 \pi D^2}{3K_{B_2}}$
CYLINDER SECTION (FORWARD TANK)	$L_2 = \frac{4k_f W_B}{\pi(1+r_m)\rho_f D^2} - \frac{D}{3} \left(\frac{1}{K_{E_2}} + \frac{1}{K_{B_2}} \right)$	$S_{L_2} = \frac{4k_f W_B}{(1+r_m)\rho_f} + \frac{\pi D^2}{3} \left(\frac{1}{K_{E_2}} + \frac{1}{K_{B_2}} \right)$
FORWARD BULKHEAD (FORWARD TANK)	$E_2 = \frac{D}{2K_{E_2}}$	$S_{E_2} = \frac{0.52 \pi D^2}{3K_{E_2}}$
FORWARD SKIRT	$e_2 = E_2$	$S_{e_2} = \frac{\pi D^2}{2K_{E_2}}$
SUCTION LINE (AFT TANK)	(AXIAL LENGTH) $H_1 = \frac{1}{128} \left\{ D \frac{\pi}{n} \right\}^{1/3}$	
SUCTION LINE (FORWARD TANK)	(AXIAL LENGTH) $H_2 = \frac{1}{128} \left\{ D \frac{\pi}{n} \right\}^{1/3} + 0.06 \frac{D}{2} \left[K_{E_1} + K_{B_2} \right] + \frac{D}{6} \left[\frac{1}{K_{E_2}} + \frac{1}{K_{B_1}} \right] + \frac{4}{\pi} \frac{k_o r_m}{(1+r_m)\rho_O} \frac{W_B}{D^2}$	
STAGE	(TOTAL LENGTH) $L_t = \frac{1}{128} \left\{ D \frac{\pi}{n} \right\}^{1/3} + \frac{4}{\pi} \frac{k_o r_m}{D^2 (1+r_m)} \left\{ \frac{k_f}{\rho_f} + \frac{W_B}{\rho_O} \right\} + \frac{D}{2} \left\{ 0.06 (K_{E_1} + K_{B_2}) + \frac{1}{3} \left[\frac{1}{K_{B_1}} + \frac{1}{K_{E_1}} + \frac{1}{K_{B_2}} + \frac{1}{K_{E_2}} \right] \right\}$	



These equations are applicable for fuel in forward tank and for propellant loading conditions of

$$W_B \geq \frac{\rho_f}{k_f} (1+r_m) \frac{\pi}{12} D^3 \left(\frac{1}{K_{E_2}} + \frac{1}{K_{B_2}} \right)$$

and

$$W_B \geq \frac{\rho_O (1+r_m)}{k_O r_m} \frac{\pi}{12} D^3 \left(\frac{1}{K_{E_1}} + \frac{1}{K_{B_1}} \right)$$

Range of spheroidal bulkhead shape is

$$1/2 \leq K_{B_1} \leq 2$$

and

$$1/2 \leq K_{E_1} \leq 2$$

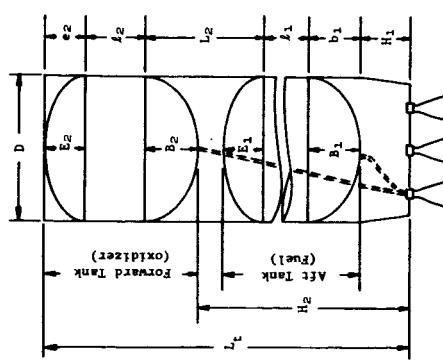
where

$$K_{B_1} = \frac{D}{2B_1}$$

$$K_{E_1} = \frac{D}{2E_1}$$

FIGURE 8. STAGE CONFIGURATION, FUEL IN FORWARD TANK

COMPONENT	LENGTH	SURFACE AREA
BOATTAIL	$H_1 = \frac{1}{128} \left\{ D \frac{\bar{E}}{n} \right\}^{1/3}$	$S_{H_1} = \pi \frac{D^{4/3}}{128} \left\{ \frac{\bar{E}}{n} \right\}^{1/3}$
AFT SKIRT	$b_1 = B_1$	$S_{b_1} = \frac{\pi D^2}{2B_1}$
AFT BULKHEAD (AFT TANK)	$B_1 = \frac{D}{2K_{B_1}}$	$S_{B_1} = 0.52 \frac{\pi D^2}{2K_{B_1}}$
CYLINDER SECTION (AFT TANK)	$\ell_1 = \frac{4k_f}{\pi(1+r_m)} \rho_o D^2 - \frac{D}{3} \left(\frac{1}{K_{E_1}} + \frac{1}{K_{B_1}} \right)$	$S_{\ell_1} = \frac{4k_f}{(1+r_m)} \rho_o D + \frac{\pi D^2}{3} \left(\frac{1}{K_{E_1}} + \frac{1}{K_{B_1}} \right)$
FORWARD BULKHEAD (AFT TANK)	$E_1 = \frac{D}{2K_{E_1}}$	$S_{E_1} = 0.52 \frac{\pi D^2}{2K_{E_1}}$
INTERTANK	$L_2 = \frac{D}{2K_{E_1} K_{B_2}} \left[K_{B_2} (1 + 0.06 K_{E_1}^2) + K_{E_1} (1 + 0.06 K_{B_2}^2) \right]$	$S_{L_2} = \frac{\pi D^2}{2K_{E_1} K_{B_2}} \left[K_{B_2} (1 + 0.06 K_{E_1}^2) + K_{E_1} (1 + 0.06 K_{B_2}^2) \right]$
AFT BULKHEAD (FORWARD TANK)	$B_2 = \frac{D}{2K_{B_2}}$	$S_{B_2} = \frac{0.52 \cdot \pi D^2}{2K_{B_2}}$
CYLINDER SECTION (FORWARD TANK)	$\ell_2 = \frac{4k_o r_m}{\pi(1+r_m)} \rho_o D^2 - \frac{D}{3} \left(\frac{1}{K_{E_2}} + \frac{1}{K_{B_2}} \right)$	$S_{\ell_2} = \frac{4k_o r_m}{(1+r_m)} \rho_o D + \frac{\pi D^2}{3} \left(\frac{1}{K_{E_2}} + \frac{1}{K_{B_2}} \right)$
FORWARD BULKHEAD (FORWARD TANK)	$E_2 = \frac{D}{2K_{E_2}}$	$S_{E_2} = \frac{0.52 \cdot \pi D^2}{2K_{E_2}}$
FORWARD SKIRT	$e_2 = E_2$	$S_{e_2} = \frac{\pi D^2}{2K_{E_2}}$
SUCTION LINE (AFT TANK)	(AXIAL LENGTH) $H_1 = \frac{1}{128} \left\{ D \frac{\bar{E}}{n} \right\}^{1/3}$	
SUCTION LINE (FORWARD TANK)	(AXIAL LENGTH) $H_2 = \frac{1}{128} \left[D \frac{\bar{E}}{n} \right]^{1/3} + 0.06 \frac{D}{2} \left[K_{E_1} + K_{B_2} \right] + \frac{D}{6} \left[\frac{1}{K_{E_1}} + \frac{1}{K_{B_2}} \right] + \frac{k_f}{\pi (1+r_m)} \rho_o \frac{D^2}{2}$	
STAGE	(TOTAL LENGTH) $L_t = \frac{1}{128} \left[D \frac{\bar{E}}{n} \right]^{1/3} + \frac{4}{\pi} \frac{\rho_o}{D^2} \frac{H_1}{(1+r_m)} + \frac{D}{2} \left\{ 0.06 (K_{E_1} + K_{B_2}) + \frac{1}{2} \left[\frac{1}{K_{B_1}} + \frac{1}{K_{E_2}} + \frac{1}{K_{B_2}} \right] \right\}$	



These equations are applicable for oxidizer in forward tank and for propellant loading conditions of

$$W_B \geq \frac{\rho_f}{k_f} (1+r_m) \frac{\pi}{12} D^3 \left(\frac{1}{K_{E_1}} + \frac{1}{K_{B_2}} \right).$$

and

$$W_B \geq \frac{\infty (1+r_m)}{k_o r_m} \frac{\pi}{12} D^3 \left(\frac{1}{K_{E_2}} + \frac{1}{K_{B_2}} \right).$$

Range of spheroidal bulkhead shape is

$$1/2 \leq K_{B_1} \leq 2$$

and

$$1/2 \leq K_{E_1} \leq 2$$

where

$$K_{B_1} = \frac{D}{2B_1}$$

$$K_{E_1} = \frac{D}{2E_1}$$

FIGURE 9. STAGE CONFIGURATION, FUEL IN AFT TANK

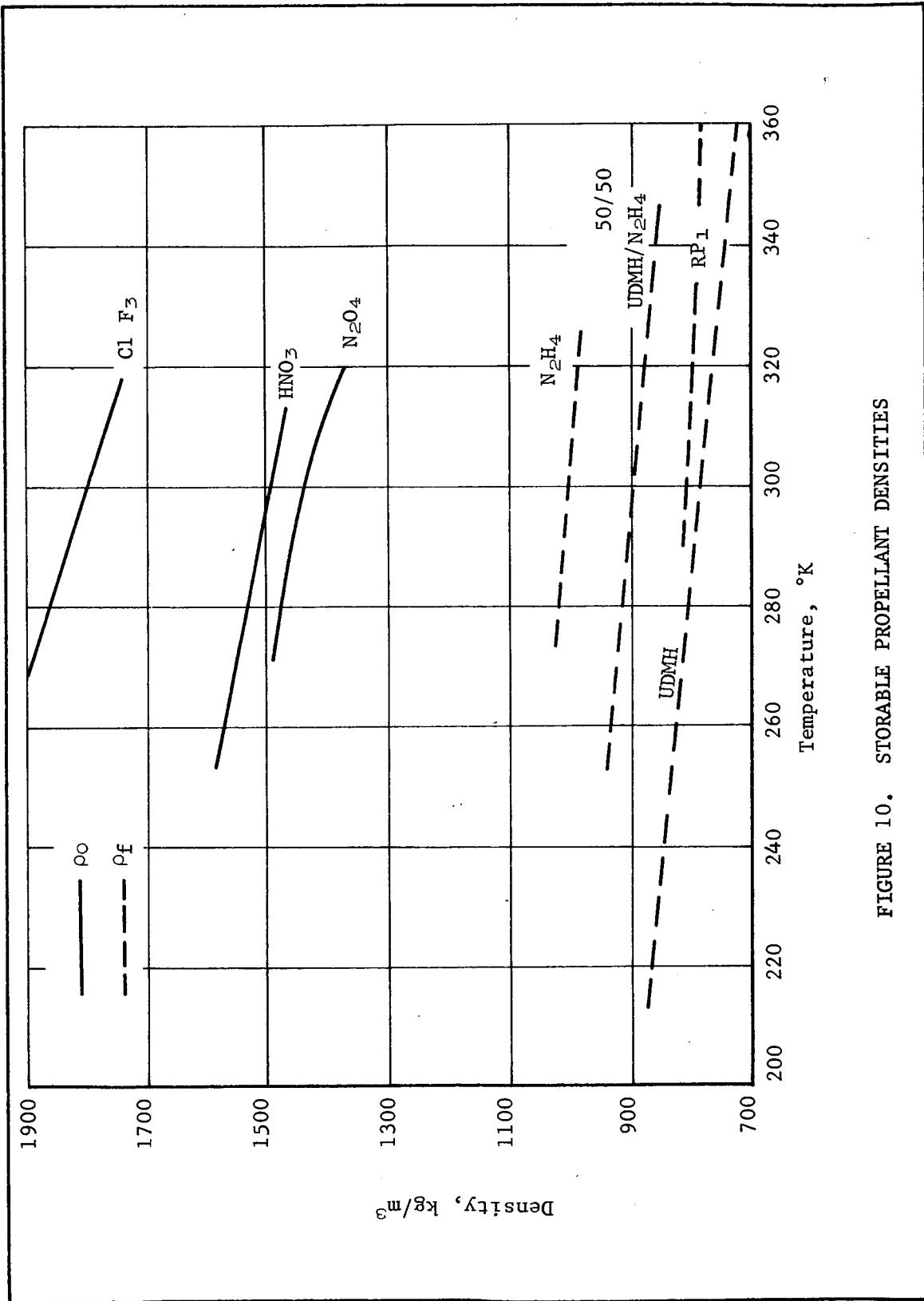


FIGURE 10. STORABLE PROPELLANT DENSITIES

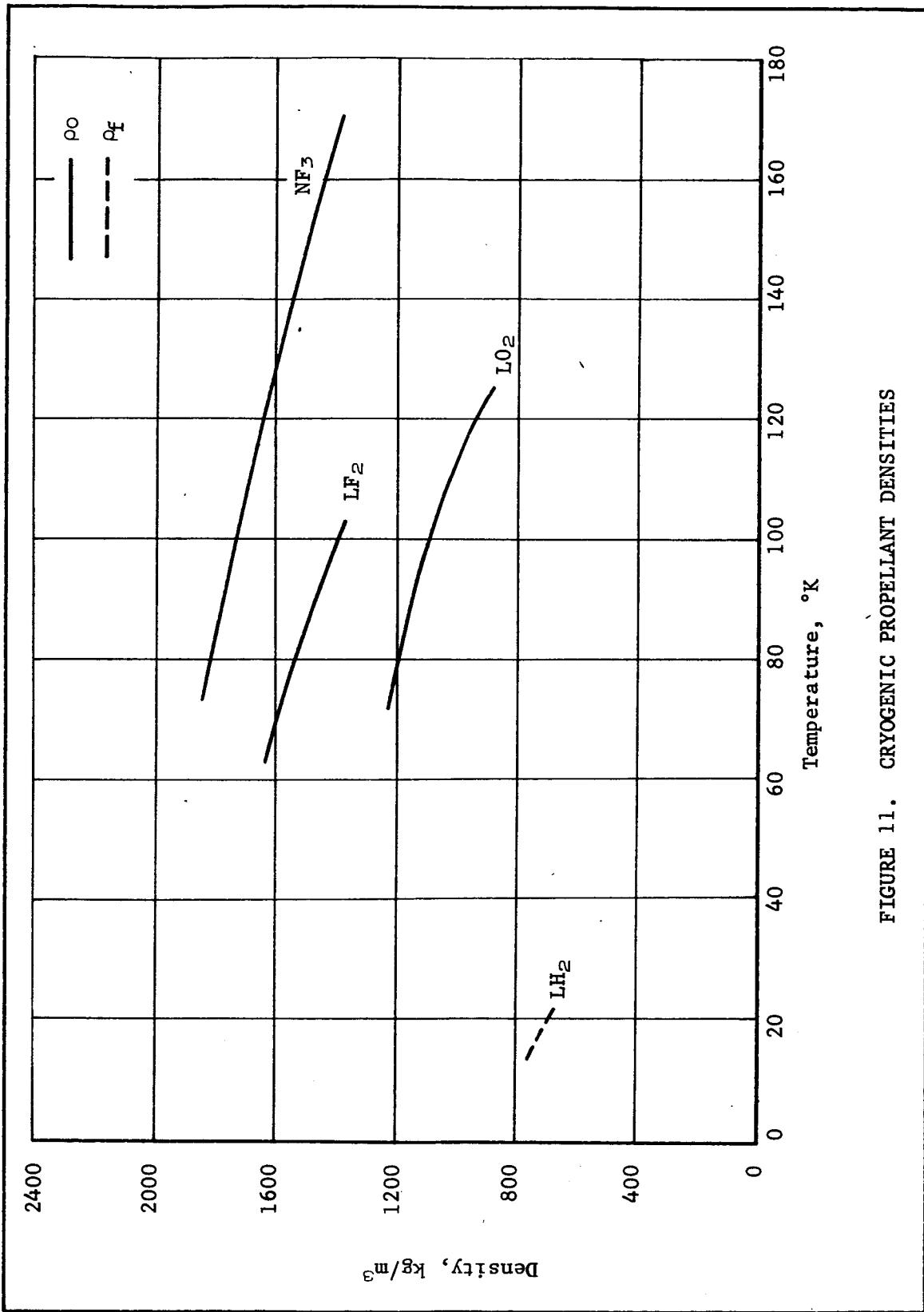


FIGURE 11. CRYOGENIC PROPELLANT DENSITIES

Other major components of the booster stage, not here discussed, will be encountered in the mass analysis following Part I of this series. With the exception of main propulsion engines, the relative size and location of remaining components will not influence the stage configuration of Figures 8 and 9. On the other hand, input parameters to the stage configuration presented here will affect the size, mass, and perhaps location of components and items that follow.

7.0 Conclusion

It has been demonstrated that, through a reasonable understanding of the relative importance of each major component, a critical selection of parameters, a parametric formulation of its geometry, and a logical summation of all these component equations can yield a mathematical model which adequately describes the booster configuration. In a similar approach, the mathematical mass model can be derived.

It was interesting to note in paragraph (6.0) that once the stage geometry was mathematically assembled, the input design parameters could be arbitrarily varied and the consequential stage geometric variations instantly obtained with as much validity as a graphic layout.

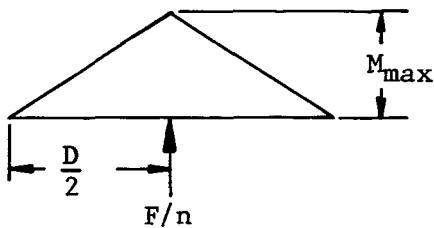
Another interesting feature about this mathematical design is that, having developed the necessary parametric equations of significant components, components may be readily rearranged or specific components may be substituted or eliminated to essentially design another type vehicle. This flexibility is analogous to a "building set," and the accumulation and cataloging of properties in parametric form for the various components applicable to stage design are encouraged.

It may also be concluded that this mathematical booster design method may be extended to other stages and developed into a subroutine to be admitted into trajectory and performance programs. Then, through the interaction of stage design parameters serving as inputs to the trajectory analysis and vice versa, optimum performance, trade-offs, and trends may be established in the early phases of preliminary analysis.

APPENDIX
THRUST STRUCTURE DEPTH PARAMETRIC ANALYSIS

A very common type of thrust structure used for mounting inboard engines of multi-engine arrangements is a system of beams, or trusses, supported by the boattail skin. The outboard engines may be symmetrically mounted on the boattail periphery. If we examine a diametrical beam supporting a single, center engine, we note that the beam

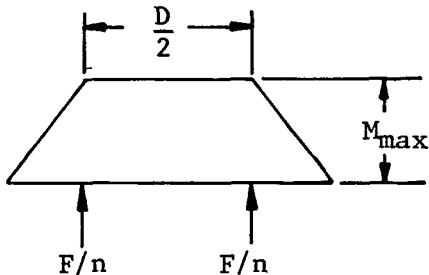
is nearly simply supported by the flexible boattail skin, the beam length is approximately that of the stage diameter (D), and the maximum induced bending moment occurs at mid-span of the beam having a magnitude of



$$M_{\max} = \left(\frac{F}{n}\right) \frac{D}{4} .$$

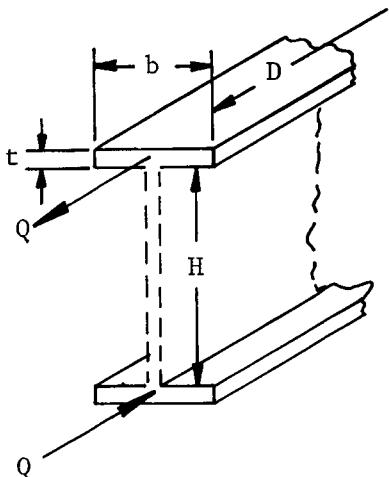
If we mount two inboard engines on this diametrical beam, a probable arrangement would position these engines a distance of ($D/2$) apart. The maximum moment would then be approximately equal to that

of the single, center engine. This externally applied moment, which is essentially a function of the applied thrust loads and the geometry of the thrust structure, may be expressed in the more general form



$$M_e = k_2 \left(\frac{F}{n}\right) D . \quad (a)$$

The "external moment" must be balanced by the "internal moment" of the thrust structure through its geometric and material properties. If we represent the structure as a built-up beam and make the simplifying assumption that the web is designed to satisfy the shear loads while the flanges satisfy the bending moment, we may express the beam internal moment as a couple having an arm of (H) length and load of (Q) or $M_a = HQ$. The maximum allowed couple load which this flange may resist



is the product of the allowable material strength and the cross sectional area of the flange. Substituting into the above couple equation (M_a) we get $M_a = H\sigma_a t b$. Since upper and lower flanges are subjected to compressive stresses, either while on the pad or during flight conditions, the flanges behave as longitudinally ribbed plates whose critical buckling stress [3] is

$$\sigma_{cr} = \frac{t^2}{b^2} k_3 \frac{E}{(1 - \nu^2)}$$

or solving for the thickness we get $t \approx k_4 b$. But the flange width (b) of optimal beam designs bear a relationship to the height (H) so that $b = k_5 H$. Substituting these relationships of (H) for (t) and (b) into the internal moment equation leads to

$$M_a = H\sigma_a [k_4 k_5 H] [k_5 H]. \quad (b)$$

Equating the external and internal moments of equations (a) and (b), respectively, and combining constants of proportionality, we obtain the expression presented on page 21, which is

$$D \frac{\bar{F}}{n} = k_1 \sigma H_1^3. \quad (c)$$

An identical expression would have resulted had the thrust structure been represented as a truss.

REFERENCES

1. Blumrich, J. F., "Multi-Cell Structure Studied for Large Boosters," Space/Aeronautics, p. 86, January 1963.
2. Blumrich, J. F., "Semi-Toroidal Tank," Astronautics and Aero-nautics, p. 64, February 1964.
3. Timoshenko, S., Theory of Elastic Stability, McGraw-Hill, New York, First Edition, 1936.

BOOSTER PARAMETRIC DESIGN METHOD FOR
PERFORMANCE AND TRAJECTORY ANALYSES

PART I: CONFIGURATION

V. VERDERAIME

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